**Linear Programming**

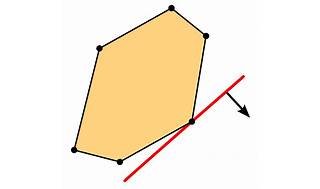
**Linear programming (LP)**, also known as **linear optimization**, is an optimization technique used to achieve the best outcome by maximizing or minimizing a linear objective function while adhering to linear equality and inequality constraints. Let’s break it down:

1. **Objective Function**: Linear programming begins with an **objective function** that defines the quantity we want to optimize (e.g., maximize profit or minimize cost). This function is linear, meaning it involves variables multiplied by constants and summed together.
2. **Constraints**: The constraints represent the limitations or conditions we must satisfy. These constraints are also linear, expressed as linear inequalities or equalities involving the same variables as the objective function.
3. **Feasible Region**: The feasible region is the intersection of half-spaces defined by the constraints. It forms a convex polytope (a multi-dimensional polygon) in the variable space.
4. **Optimization**: The goal is to find a point within the feasible region where the objective function reaches its maximum (or minimum) value.

**Graphical method of solving linear**

**Programming problems**

1. **Formulate the Problem**: Start by creating a mathematical representation of the problem. Translate the given constraints into equations or inequalities. You’ll have an **objective function** (usually to maximize profit or minimize cost) and constraints.
2. **Graph the Constraints**: Plot the system of constraints on a coordinate plane. This will define the **feasible region**, which is the intersection of all constraints. Each constraint corresponds to a boundary line or curve.



1. **Identify Corner Points (Vertices)**: Locate the **corner points** of the feasible region. These points occur where the constraint lines intersect. Each corner point represents a specific combination of decision variables.
2. **Evaluate the Objective Function**: Substitute the coordinates of each corner point into the objective function. Determine which corner point optimizes the objective (maximizes or minimizes it).
3. **State the Solution**: The corner point that yields the best objective value is your solution. If you’re maximizing profit, it’s the highest value; if minimizing cost, it’s the lowest value.

**Linear Programming: Farmer’s Example**

Suppose we have a farmer who wants to maximize their profit by planting two crops: **wheat** and **corn**. The farmer has limited resources: **fertilizer (F)** and **pesticide (P)**. Here are the details:

1. **Crop Information**:
   * Wheat: Yields 25 bushels per acre, requires 10 hours of work per week.
   * Corn: Yields 10 bushels per acre, requires 4 hours of work per week.
   * Wheat sells for $4 per bushel, and corn sells for $2.50 per bushel.
2. **Resource Constraints**:
   * The farmer has 7 acres of land and 40 hours of work per week.
   * Fertilizer requirement for wheat: F1 kilograms per square kilometer.
   * Pesticide requirement for wheat: P1 kilograms per square kilometer.
   * Fertilizer requirement for corn: F2 kilograms per square kilometer.
   * Pesticide requirement for corn: P2 kilograms per square kilometer.
3. **Decision Variables**:
   * Let (x1) represent the number of acres of wheat planted.
   * Let (x2) represent the number of acres of corn planted.
4. **Objective Function (Maximize Profit)**:
   * We want to maximize the profit function: [ Z = 4x1 + 2.5x2]
5. **Constraints**:
   * Labor constraint: (10x1 + 4x2 = 40) (hours of work)
   * Land constraint: (x1 + x2 = 7) (acres of land)
   * Fertilizer constraint: (F1x1 + F2x2 =F) (fertilizer availability)
   * Pesticide constraint: (P1x1 + P2x2 =P) (pesticide availability)
6. **Non-Negativity Constraints**:
   * (x1 >= 0)
   * (x2 >= 0)

**Complete Linear Programming Model:**

Maximize (Z = 4x1 + 2.5x2)

**Feasible Solution Example:**

* Let’s say:
  + (x1 = 5) acres of wheat
  + (x2 = 10) acres of corn
* Profit ((Z)) = $700
* Labor constraint check: (10(5) + 4(10) = 40) hours (within limit)
* Land constraint check: (5 + 10 = 15) acres (within limit)